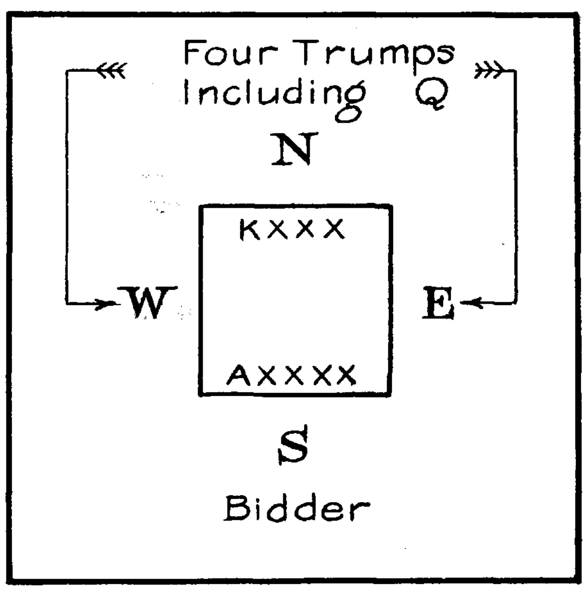
The Binomial Theorem in Bridge

By SAMUEL FREIFIELD, L-I.

The kind of bridge which I am discussing in this article differs materially from the kind in which most engineers find their interest, yet from the social aspect bridge, the card game, merits a little attention. Particularly within the past 25 years has the game gained in popularity. I will not dwell on its history here; suffice it to say that skill in playing the game has its rewards—to play bridge well seems to be a social requisite.

Professional men have generally been criticized for being engrossed in their respective professions to the extent that they care not one whit for the lighter aspects of life. Socially, they are interminable bores, so the critics say. It must be conceded, too, that there is some degree of truth in this sweeping indictment. Engineers, for example, seem to get their greatest pleasure in discussing highly technical subjects—and this at all times. So it is with chemists, biologists, and others trained as they are in their respective fields which demand the highest kinds of specialization. How many wheezes we hear on this subject! A famous scientist recently declared that the energy requirement for the production of human speech is extremely low, yet I venture to say, that if all the energy expended in giving vent to such jokes were properly converted, this city could be supplied with electrical energy for a century or so.



The problem of trump distribution. Will South get his opponent's Queen on the second lead?

Yet, if we concede that there is some measure of truth in what the critics say, surely remedial measures are in order.

Bridge, while a popular game from the social standpoint, happens to be of such a nature that it lends itself readily, in some respects, to mathematical analysis. If only for this reason engineers ought to become very proficient, by virtue of their training, in acquiring the skill necessary to play the game well.

For the most part, those who have contributed to the theory of probability have not concerned themselves with trying to apply the various principles to card games, and rightly so. Such attempts are *per se* complex in nature. The trained man, and *a fortiori* the layman, is at a loss in applying such principles. In various phases of this particular card game, however, the application of some of the elementary principles can easily be undertaken. I will consider, in the following, the use of the bionomial theorem in a few situations that arise in the game of bridge.

(1) Suppose A and B are opposed to X and Y. A receives the bid, and B exposes his hand. B shows 4 trumps, say diamonds, with the King high. A has 5 diamonds Ace high, and the Queen is out against him; in the hand of either of his opponents. A, knowing that there are in the aggregate 13 trumps, is aware that X and Y have, together, 4 trumps, one of which is the Queen. A is confronted with this dilemma: Shall he call for trumps, assuming that the four trumps held by X and Y together are evenly divided, and if this be so, "kill" the Queen on the second trick, taking it with his King, or shall he play his hand under the assumption that there may be a 3-1 or a 4-0 division of trump between X and Y? It would appear, superficially, that there is more chance of an even (2-2) division, but is there?

We now apply an elementary principle in probability, together with the binomial theorem: If a is the chance that X has a certain card (of the four that X and Y have together), and b is the chance that Y has that card, obviously $a+b=1$.

Since we are considering 4 cards in the hands of X and Y, together, we raise $(a+b)$ to the fourth power:

$$(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4.$$

Then since a^2b^2 satisfies the condition that there be an even distribution (2-2) of the 4 cards, there are six ways in which this may happen. The total number of coefficients is $1+4+6+4+1$ which equals 16, hence there are 6 chances out of 16 that there will be an even distribution of the four cards in question. A will play his hand accordingly.

(2) No matter what the distribution is, the same method can be applied. Thus, if X and Y have 5 trumps together, and it is a question, say, whether either has all 5. The solution will be as follows:

$$(a+b)^5=a^5+5a^4b+10a^3b^2+a^2b^3+5ab^4+b^5$$

The coefficients are $1+5+10+10+5+1=32$.

(Incidentally, to simplify this phase of the matter the total of the coefficients can always be determined by raising 2 to the appropriate power: thus $2^5=32$, and in the previous example $2^4=16$). Then the chances are $1(a^5)+1(b^5)=2$, or 2 out of 32 for the distribution as noted.

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Such applications need not be confined to the trumps. They are just as pertinent with regard to the other suits, and the same types of analysis may accordingly be employed.

It might be argued that this is an over-simplification of the phase of bridge about which I have spoken, for I have not considered the method and technique of bidding, before actual card play commences. Granting that this is so, I can hardly see where such analysis will militate against the user; where such user carefully takes into consideration, as he should, the bidding that has preceded the actual play he will find more and more necessary applications for the principle as suggested.